Recommended Method of Finding the Natural Power Factor

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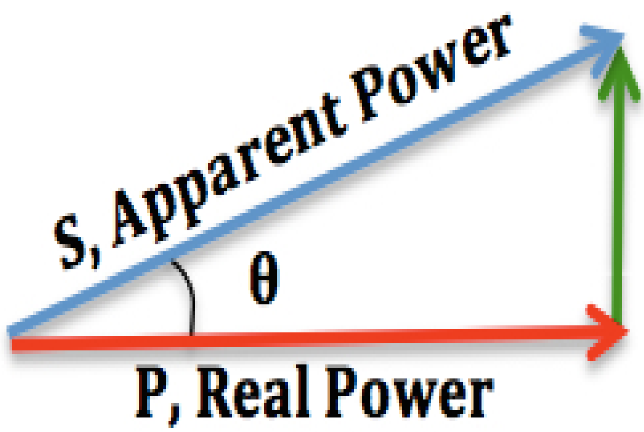

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1 Executive Summary

The power factor of a AC system is the ratio of active and apparent power, and indicates the ability of the electrical system to do useful work. Electrical utilities want high power factor to increase asset utilization and deliver more energy. To do so they typically add in capacitors to compensate inductive loads or inductors to compensate capacitive loads.

The natural power factor (NPF) is the power factor that the system would have without any compensation. This is used for planning purposes in the power system, and if incorrect can cause unacceptable performance, equipment overload and additional costs.

There are many different methods of calculating the NPF. This paper concludes that the most consistent year over year method is the PQ Plot with $NPF(P_{avg})$ as it has the lowest standard deviation of 0.0033. This method involves plotting P vs Q_{uncomp} and finding the line of best fit to get Q_{uncomp} as a function of P. The NPF can then be written as a function of P, then the NPF is taken to be $NPF(P_{avg})$. The $NPF(P_{avg})$ also provides the most conservative NPF. The method that gives the NPF that is likely most consistent with the NPF at peak loading the $NPF(P_{max})$. All methods are summarized in figure 8.

The PQ plot methods also have another advantage. The natural power factor is not a single number and changes as the loading changes. Part of this method is deriving a functional relationship between NPF and P, this means that for any P the NPF for can be calculated giving a more precise NPF for a given scenario.

2 Introduction

The purpose of this report is to summarize different options for calculating the natural power factor and objectively determine the best method that gives the most consistent year over year natural power factor at system peak.

Power Triangle

In an AC system both current($\underline{i}$) and voltage($\underline{v}$) are represented as phasors. Phasors are complex numbers that represent the amplitude and phase angle of a sinusoidal wave. Since power($\underline{S}$) is current times voltage ($\underline{S} = \underline{v}\underline{i}$), then it is also represented in the complex plane. This means there is some power that is on the imaginary axis, this is called 'reactive power(Q)' and is indicated with units of MVAr. There is also some power on the real axis, which is called 'active power(P)' and is indicated with units of MVA. The total power, called the 'apparent power($\underline{S}$)' is the vector addition of the real and reactive powers $\underline{S} = P + jQ$, where j is the imaginary unit. This relationship can be seen in figure 1.

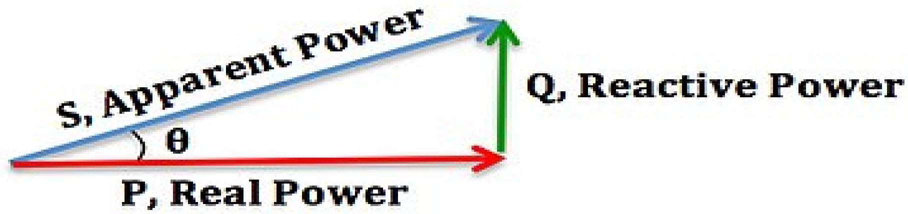

Figure 1: Power Triangle

The phase difference between voltage and current comes from the type of load being supplied. The load has some impedance($\underline{Z}$) which is made up of a resistance(R) and reactance(X), $\underline{Z} = R + jX$. The reactance has some capacitive($X_C = \frac{1}{\omega C}$) and inductive($X_L = \omega L$) components, where ω is the angular frequency ($\omega = 2\pi f$).

$$X = X_L - X_C \tag{1}$$
$$= \omega L - \frac{1}{\omega C} \tag{2}$$

Thus P and Q can be rewritten in terms of the impedance.

$$P = I^2 R \tag{3}$$
$$Q = I^2 X \tag{4}$$
$$= I^2 (X_L - X_C) \tag{5}$$

When $X > 0$ then the the total reactance is said to be inductive, and when $X < 0$ the total reactance is said to be capacitive.

Power Factor (PF)

The power factor (PF) is the ratio of active and apparent power. This ratio indicates the ability of the electrical system to do useful work.

$$PF = \frac{\text{Active Power}}{\text{Apparent Power}} = \frac{P}{S} \tag{6}$$

Low power factors limit the capacity of an electrical system to deliver energy, and can contribute power quality issues. A good power factor is typically from 0.95 to 1.0 at system peak.

When the total reactance is inductive ($X > 0$) then the power factor is said to be lagging, and when the total reactance is capacitive ($X < 0$) the power factor is said to be leading. These can be seen in figures 2 and 3.

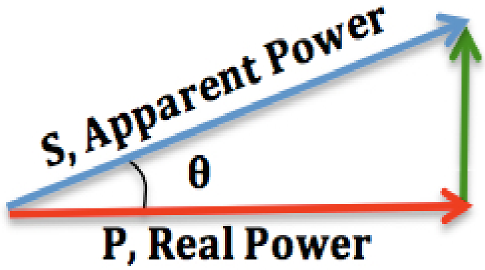

Figure 2: Lagging Power Factor

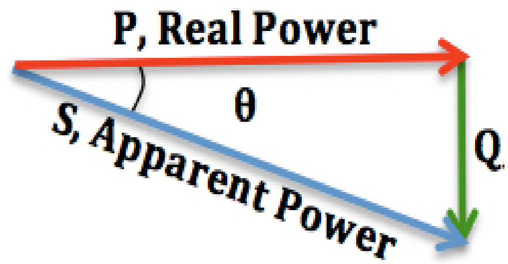

Figure 3: Leading Power Factor

Natural Power Factor (NPF)

Electrical utilities want to decrease the magnitude of their systems impedance to achieve a good power factor. They do so by providing reactive compensation (X_{comp}) causing the total impedance in their system to become $X_{total} = X_L - X_C + X_{comp}$. Typical systems have a lagging power factor ($X_L - X_C > 0$) so they provide capacitor compensation meaning that X_{comp} is a negative value effectively reducing the magnitude of the total impedance. The natural power factor (NPF) is the power factor if there were no compensation in place.

$$NPF = \frac{\text{Active Power}}{\text{Uncompensated Apparent Power}} \tag{7}$$

$$= \frac{P}{S_{uncomp}} \tag{8}$$

$$= \frac{P}{\sqrt{P^2 + Q_{uncomp}^2}} \tag{9}$$

Where

$$Q_{uncomp} = I^2 X_{uncomp} = I^2 (X_{total} - X_{comp}) \tag{10}$$

The NPF methods tested all provide a single value for the NPF over a study period that can be used by planners to make decisions about future system needs, capacity upgrade timing and required compensation support.

3 Calculation Methods

There are multiple different methods to calculate the natural power factor (NPF) which are described below. Each method uses 15 minute PI data with both active and reactive power information as well as station capacitor service data. The NPF for each 15 minute interval is calculated using equation 9. The data is taken over span of 4 months in the winter (Nov-Feb), and 4 months in the summer (May-Aug) as the study times to get a single value for the summer and winter NPF using the following methods:

1. Average: Take the NPF as the average of all the NPF's for each 15 minute interval over the study time.
2. Maximum: Take the NPF corresponding to the single maximum point of the active power.

3. Bin Histogram: Create a histogram of all the NPF over the study time with bin spaces of 0.01. Takes the NPF as the most frequent occurring value. An example histogram is shown in figure 4, in this case the NPF would be 0.94.

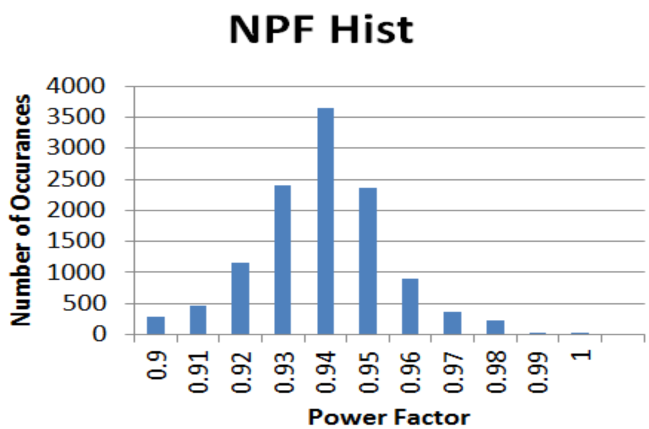

Figure 4: NOR Summer FY2018 NPF Histogram

4. 60% Above: Take the NPF that has at least 60% of the NPF's above it. An example plot is shown in figure 5, in this case the NPF would be 0.98.

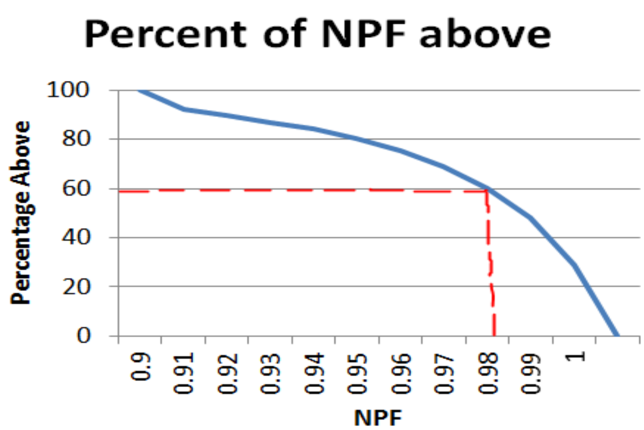

Figure 5: CSQ Winter F2018 % Above Plot

5. 20% Data Filter Average: Take the average of the NPF's that have corresponding active power within 20% of the peak active power.

6. PQ Plot: Plot P vs Q_{uncomp}, find the line of best fit and get Q_{uncomp} as a function of P, call this $Q_u(P)$. The NPF can then be written as a function of P using equation 9:

$$NPF(P) = \frac{P}{\sqrt{P^2 + Q_u(P)^2}} \tag{11}$$

From here there were 2 methods tested:

- Take the NPF to be $NPF(P_{max})$
- Take the NPF to be $NPF(P_{avg})$

Figure 6 gives an example of a PQ plot. In this case $Q_u(P) = 0.1961P - 19.529$.

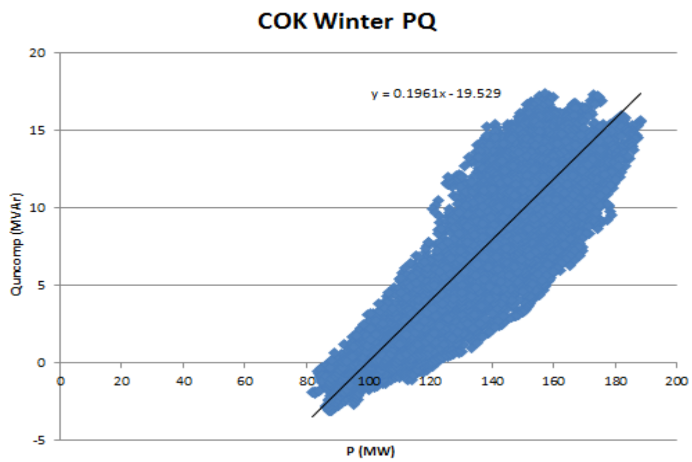

Figure 6: COK Winter F2018 PQ_{uncomp} Plot

4 Analysis

To analyze each method, 8 BC Hydro substations with good PI data and no feeder capacitor compensation were selected at random. Stations with feeder capacitors were not selected as there is no in service data for feeder capacitors, thus no way to accurately determine whether feeder compensation is in or out. The NPF of each of these substations was computed with each method every year for the past 3 years.

The average NPF for each method was computed by taking the average NPF over the 3 years for each substation for both winter and summer. These can be compared to determine how conservative each method is (more conservative meaning a lower NPF) and how much they vary from one another.

To test the consistency of a method the standard deviation was found from the NPF's produced by that method for each substation over the 3 years. For each season (winter and summmer) the average of the standard deviations for each substation was taken to be the seasonal standard deviation for that method. Finally the average of the seasonal standard deviations (winter and summer) was taken to be the final standard deviation of that method. This was done for each method.

Both average and standard deviations of the NPF's for each method are summarized in figure 7. More detailed data can be found in the appendix.

#	Method	Avg NPF	Avg Std. Dev
1	Average	0.969	0.0037
2	Maximum	0.980	0.0045
3	Bin Hist	0.977	0.0060
4	60% above	0.974	0.0039
5	20% filter Avg	0.979	0.0038
6	PQ plot NPF(Pmax)	0.980	0.0041
7	PQ plot NPF(Pavg)	0.973	0.0033

Figure 7: Average standard deviation of each tested NPF method

5 Results

Figure 8 summarizes each method. Confidence levels were selected by considering the methods pros and cons as well as average NPF they produced and associated standard deviation.

#	Method	Description	Pros	Cons	Avg NPF of all tested Stations	Avg Std Dev of those NPF's	Confidence in NPF
1	Average	Take the NPF as the average of all the NPF's for each 15 minute interval over the study time.	• Conservative value • Easy to calculate • Relatively consistent year over year	• May not give NPF at peak loading	0.969	0.0037	Mid
2	Maximum	Take the NPF corresponding to the single maximum point of the active power.	• Easy to calculate • Gives the NPF at peak loading	• Considers only a single data point, may be an outlier or bad data • Not as consistent year over year	0.980	0.0045	Low
3	Bin Histogram	Create a histogram of all the NPF over the study time with bin spaces of 0.01. Takes the NPF as the most frequent occurring value. (see figure 4)	• Expresses the power factor that occurs most frequently	• May not give NPF at peak loading • Least consistent year over year • Harder to calculate	0.977	0.0060	Low
4	60% Above	Take the NPF that has at least 60% of the NPF's above it. (see figure 5)	• Conservative value • Relatively consistent year over year	• Harder to calculate	0.974	0.0039	High
5	20% filter avg	Take the average of the NPF's that have corresponding active power within 20% of the peak active power.	• Relatively consistent year over year • Skews data to get NPF closer to peak loading	• Harder to calculate • If the peak is a large outlier may only use a little amount of data	0.979	0.0038	Mid
6	PQ plot NPF(Pmax)	Plot P vs Quncomp (Qu) and find the line of best fit to get Qu as a function of P. The NPF can then be written as a function of P, then the NPF is taken to be NPF(Pavg). (see figure 6)	• Gives a NPF very close to the maximum method but more consistent year over year • Gives the NPF at peak loading • Relatively consistent year over year	• Harder to calculate	0.980	0.0041	High
7	PQ plot NPF(Pavg)	Same as the above method but take NPF to be NPF(Pmax)	• Most consistent method • Conservative value	• Harder to calculate	0.973	0.0033	High

Figure 8: Method Summary

6 Recommendation

Based on this analysis the PQ Plot NPF(P_{max}) method should be used. It gives a relatively consistent year over year NPF, and gives the NPF at the peak loading which is what is most important.

The PQ plot method also has another advantage. The NPF is not a single number and changes as the loading changes. Part of this method is deriving a functional relationship between NPF and P, this means that for any P the NPF for can be calculated giving a more precise NPF for a given scenario.

The only drawback to this method is that it takes a lot of computation to arrive at the answer. For a less accurate but also less computationally intensive method the average method should be chosen, it also gives a more conservative NPF.

Appendix

Station	Method	F16	NPF F17	F18	Avg NPF	NPF Std Dev
FRC	Average	0.914	0.918	0.901	0.911	0.0088
	Maximum	0.976	0.973	0.970	0.973	0.0032
	Bin Hist	0.900	0.900	0.900	0.900	0.0000
	60% above	0.910	0.920	0.900	0.910	0.0100
	20% filter Avg	0.967	0.966	0.963	0.966	0.0022
	PQ plot NPF(Pmax)	0.978	0.978	0.974	0.977	0.0024
	PQ plot NPF(Pavg)	0.926	0.925	0.911	0.920	0.0086
NOR	Average	0.970	0.968	0.973	0.971	0.0028
	Maximum	0.966	0.966	0.977	0.970	0.0065
	Bin Hist	0.970	0.980	0.990	0.980	0.0100
	60% above	0.970	0.960	0.970	0.967	0.0058
	20% filter Avg	0.971	0.974	0.973	0.972	0.0014
	PQ plot NPF(Pmax)	0.971	0.975	0.975	0.974	0.0021
	PQ plot NPF(Pavg)	0.971	0.970	0.975	0.972	0.0028
COK	Average	0.998	0.998	0.999	0.998	0.0003
	Maximum	1.000	0.997	0.998	0.998	0.0017
	Bin Hist	0.960	1.000	1.000	0.987	0.0231
	60% above	1.000	1.000	1.000	1.000	0.0000
	20% filter Avg	0.997	0.997	0.998	0.997	0.0005
	PQ plot NPF(Pmax)	0.999	0.996	0.996	0.997	0.0019
	PQ plot NPF(Pavg)	0.999	0.999	0.999	0.999	0.0004
DGR	Average	0.962	0.971	0.977	0.970	0.0072
	Maximum	0.966	0.970	0.974	0.970	0.0040
	Bin Hist	1.000	0.970	0.970	0.980	0.0173
	60% above	0.960	0.970	0.970	0.967	0.0058
	20% filter Avg	0.964	0.970	0.979	0.971	0.0074
	PQ plot NPF(Pmax)	0.964	0.968	0.980	0.971	0.0081
	PQ plot NPF(Pavg)	0.963	0.971	0.978	0.970	0.0075
CSQ	Average	0.981	0.969	0.973	0.974	0.0060
	Maximum	0.997	0.998	0.999	0.998	0.0006
	Bin Hist	1.000	1.000	0.980	0.993	0.0115
	60% above	0.990	0.970	0.980	0.980	0.0100
	20% filter Avg	0.985	0.990	0.993	0.989	0.0040
	PQ plot NPF(Pmax)	0.984	0.991	0.996	0.990	0.0060
	PQ plot NPF(Pavg)	0.999	0.996	0.996	0.997	0.0018
PVL	Average	0.997	0.996	0.996	0.996	0.0006
	Maximum	0.994	0.994	0.993	0.993	0.0007
	Bin Hist	1.000	1.000	1.000	1.000	0.0000
	60% above	1.000	1.000	1.000	1.000	0.0000
	20% filter Avg	0.995	0.994	0.994	0.995	0.0006
	PQ plot NPF(Pmax)	0.995	0.993	0.994	0.994	0.0011
	PQ plot NPF(Pavg)	0.997	0.996	0.996	0.996	0.0007
PML	Average	0.995	0.997	0.996	0.996	0.0009
	Maximum	0.995	0.998	0.997	0.997	0.0015
	Bin Hist	1.000	1.000	1.000	1.000	0.0000
	60% above	1.000	1.000	1.000	1.000	0.0000
	20% filter Avg	0.994	0.996	0.994	0.995	0.0009
	PQ plot NPF(Pmax)	0.997	0.996	0.994	0.995	0.0015
	PQ plot NPF(Pavg)	0.997	0.997	0.996	0.997	0.0005
KTG	Average	0.996	0.997	0.998	0.997	0.0007
	Maximum	0.994	0.995	0.995	0.994	0.0008
	Bin Hist	1.000	1.000	1.000	1.000	0.0000
	60% above	1.000	1.000	1.000	1.000	0.0000
	20% filter Avg	0.995	0.996	0.996	0.996	0.0008
	PQ plot NPF(Pmax)	0.995	0.995	0.996	0.995	0.0007
	PQ plot NPF(Pavg)	0.996	0.997	0.998	0.997	0.0007

Figure 9: Winter NPF Details

Station	Method	F16	NPF F17	F18	Avg NPF	NPF Std Dev
FRC	Average	0.831	0.836	0.838	0.835	0.0036
	Maximum	0.941	0.919	0.966	0.942	0.0235
	Bin Hist	0.900	0.900	0.900	0.900	0.0000
	60% above	0.900	0.900	0.900	0.900	0.0000
	20% filter Avg	0.917	0.922	0.940	0.926	0.0123
	PQ plot NPF(Pmax)	0.943	0.957	0.963	0.954	0.0103
	PQ plot NPF(Pavg)	0.838	0.844	0.847	0.843	0.0042
NOR	Average	0.947	0.939	0.970	0.952	0.0161
	Maximum	0.951	0.941	0.953	0.948	0.0068
	Bin Hist	0.940	0.940	0.970	0.950	0.0173
	60% above	0.940	0.940	0.970	0.950	0.0173
	20% filter Avg	0.948	0.938	0.960	0.949	0.0112
	PQ plot NPF(Pmax)	0.948	0.942	0.956	0.949	0.0071
	PQ plot NPF(Pavg)	0.948	0.940	0.969	0.953	0.0148
COK	Average	0.994	0.996	0.996	0.995	0.0012
	Maximum	0.981	0.982	0.979	0.981	0.0017
	Bin Hist	1.000	1.000	1.000	1.000	0.0000
	60% above	1.000	1.000	1.000	1.000	0.0000
	20% filter Avg	0.989	0.991	0.990	0.990	0.0011
	PQ plot NPF(Pmax)	0.985	0.989	0.986	0.986	0.0021
	PQ plot NPF(Pavg)	0.998	0.997	0.997	0.997	0.0005
DGR	Average	0.947	0.948	0.954	0.950	0.0038
	Maximum	0.937	0.942	0.942	0.940	0.0027
	Bin Hist	0.950	0.950	0.950	0.950	0.0000
	60% above	0.950	0.950	0.950	0.950	0.0000
	20% filter Avg	0.945	0.947	0.950	0.947	0.0025
	PQ plot NPF(Pmax)	0.944	0.946	0.950	0.947	0.0034
	PQ plot NPF(Pavg)	0.947	0.948	0.954	0.950	0.0039
CSQ	Average	0.979	0.976	0.978	0.978	0.0014
	Maximum	0.996	0.998	0.996	0.997	0.0013
	Bin Hist	1.000	1.000	1.000	1.000	0.0000
	60% above	0.990	0.980	0.980	0.983	0.0058
	20% filter Avg	0.989	0.986	0.990	0.988	0.0022
	PQ plot NPF(Pmax)	0.989	0.983	0.992	0.988	0.0045
	PQ plot NPF(Pavg)	0.998	1.000	0.998	0.999	0.0008
PVL	Average	0.993	0.994	0.993	0.993	0.0004
	Maximum	0.992	0.994	0.995	0.994	0.0014
	Bin Hist	0.990	0.990	1.000	0.993	0.0058
	60% above	0.990	0.990	0.990	0.990	0.0000
	20% filter Avg	0.991	0.995	0.990	0.992	0.0025
	PQ plot NPF(Pmax)	0.990	0.992	0.991	0.991	0.0007
	PQ plot NPF(Pavg)	0.993	0.994	0.994	0.994	0.0004
PML	Average	0.993	0.996	0.986	0.992	0.0053
	Maximum	0.995	0.997	0.971	0.988	0.0142
	Bin Hist	0.990	1.000	1.000	0.997	0.0058
	60% above	0.990	1.000	0.980	0.990	0.0100
	20% filter Avg	0.988	0.996	0.980	0.988	0.0082
	PQ plot NPF(Pmax)	0.990	0.996	0.972	0.986	0.0127
	PQ plot NPF(Pavg)	0.994	0.996	0.988	0.993	0.0045
KTG	Average	0.994	0.995	0.995	0.995	0.0005
	Maximum	0.999	0.997	0.997	0.998	0.0015
	Bin Hist	0.990	1.000	1.000	0.997	0.0058
	60% above	0.990	0.990	0.990	0.990	0.0000
	20% filter Avg	0.994	0.996	1.000	0.997	0.0029
	PQ plot NPF(Pmax)	0.992	0.993	0.991	0.992	0.0007
	PQ plot NPF(Pavg)	0.994	0.995	0.995	0.995	0.0005

Figure 10: Summer NPF Details

#	Method	Avg NPF	Avg Std Dev
1	Average	0.977	0.0034
2	Maximum	0.987	0.0024
3	Bin Hist	0.980	0.0077
4	60% above	0.979	0.0036
5	20% filter Avg	0.985	0.0022
6	PQ plot NPF(Pmax)	0.987	0.0030
7	PQ plot NPF(Pavg)	0.981	0.0029

Figure 11: Winter NPF Analysis Summary

#	Method	Avg NPF	Avg Std Dev
1	Average	0.961	0.0040
2	Maximum	0.973	0.0066
3	Bin Hist	0.973	0.0043
4	60% above	0.969	0.0041
5	20% filter Avg	0.972	0.0054
6	PQ plot NPF(Pmax)	0.974	0.0052
7	PQ plot NPF(Pavg)	0.965	0.0037

Figure 12: Summer NPF Analysis Summary